Unit Lesson Plan: Modeling Pythagoras' Theorem

Grade Level: 8

Unit: Measurement

SCO: Students should be able to demonstrate an understanding of the Pythagorean relationship using models.

Prior Knowledge: Students should know how to compute the area of squares, rectangles, triangles and circles. They should also have a basic understanding of angles (acute, right and obtuse) and triangles (isosceles, equilateral, scalene and right).

Setting the Stage:

1. <u>12-knot string activity</u>

Discovery/Restructuring:

1. Manipulative presentation by Amanda Knox using squares

Closure/Wrap-up:

- 1. <u>SMARTboard manipulative</u>, adapted from Pierette Pheeney
- 2. NCTM Manipulative
- 3. Straw activity for non-right triangles
- 4. <u>Magic triangle</u>
- 5. Area of semi-circles and other shapes
- 6. Word problem application (Create their own as well)
- 7. <u>Pythagorean tree</u>
- 8. <u>Relation of Pythagorean tree to fractals</u>
- 9. Choice Board Final Project

Setting the stage

1. 12-knot string activity

Objectives:

- 1. To review the properties of a right triangle
- 2. Introduce students to the 3-4-5 rule and prepare them to learn Pythagoras's theory
- 3. To set the context for Pythagoras's Theory

Materials:

Each pair of students should have:

- a loop of string or ribbon with 12 equally-spaced knots
- two sticks, rulers or straws
- tape to hold sticks in place

Advanced Preparation:

- Prepare loops of string or ribbon with 12 equally-spaced knots for each pair of students. Alternatively, prepare just one to demonstrate the method to the class.
- Prepare a SMARTboard/PowerPoint presentation on how Egyptians/Carpenters used the 3-4-5 rule and review the properties of right triangles.

Instruction:

- Have shapes of square, rectangle, circle and triangle ready for students to review areas. Have them come up to SMARTboard to label diagrams. Please see <u>Appendix A</u>.
- Ask students to place the sticks at a perfect 90-degree angle using only the string as a measurement tool. Guide them through the process by suggesting that they make different shapes. Do not let this continue too long, it is just a pre-activity. Eventually have them arrive at a right triangle.
- Explain that what they found is the 3-4-5 rule. Set historical context for this rule by explaining how Egyptians used it for building the pyramids. Explain that carpenters use this rule too.
- Ask students if they know what kind of triangle is formed by the 3-4-5 rule. Review properties of a right triangle (90 degree angle, longest side called hypotenuse, etc.)
 - This is a good time to go over essential types of angles and triangles. Give students a variety of triangles and have them label hypotenuses (when appropriate). Please see <u>Appendix B</u>.

Discovery/Restructuring

1. Manipulative presentation by Amanda Knox using squares

For objective, materials and instruction, please refer to <u>manipulative lesson plan</u> by Amanda Knox.

Closure/Wrap-Up

1. SMARTboard Manipulative, adapted from Pierette Pheeney

Objectives:

- 1. Further reinforce the Pythagorean model, by having students rearrange the smaller squares to fit into the larger one
- 2. Provide a concrete, step-by-step reinforcement of the model by breaking each part into unit squares

Materials:

• Either ensure access to a SMARTboard, or provide handout in <u>Appendix C1</u> with scissors and Scotch tape.

Instruction:

- Using a SMARTboard, have the students come up to move the red and blue unit squares from the small and medium sides into the outline on the large side. It is useful to have two copies of the outline, one where the red and blue unit squares are oriented non-diagonally, and the second copy rotated so that the outline on the large side is oriented non-diagonally.
- Without a SMARTboard, simply have the students cut out the squares and tape them into the larger square.

If students need additional reinforcements, make copies of <u>Appendix C2</u>, which is also included in Amanda Knox's manipulative lesson plan. Students should be provided with a paper copy and if possible, have access to the manipulative on a SMARTboard for review with the entire class.

2. NCTM Manipulative

Objectives:

- 1. Further reinforce the Pythagorean model, by having students rearrange the smaller squares to fit into the larger one
- **2.** Less arbitrary than the cuts shown in Appendix C2 manipulative, but more general than the squares of the SMARTboard manipulative from Pierette

Materials:

Instructor should provide handout in <u>Appendix D</u>.

Instruction:

Following the first diagram, show students that the two smaller squares can be rearranged to make the larger square. The construction is as follows:

- Draw a right angle triangle. Draw a square against each of the sides.
- Take the small and medium squares, and place them side-by-side
- Draw two copies of the original right angle triangle over the squares, such that the small end of the triangle matches with one side of the small square, and the medium side of the triangle matches with one side of the medium square. See Illustration in Appendix D.
- Cut along each of the resulting lines.

Ask the students if they can rearrange the cutouts into a square. The solution is shown on the second page of <u>Appendix D</u>. However, there is another solution – that of a rectangle, that is very close to being a square. Have the students' measure the sides of their figure with a cutout of the original triangle to ensure the sides are of length c.

3. Straw activity for non-right angle triangles

Objective:

1. To have students discover that the Pythagorean relationship does not hold for *all* triangles, only for right triangles.

Materials:

Each student should have:

- Squares made of graph paper (same as the ones used in <u>Amanda's manipulative</u>)
- Several straws or sticks with lengths corresponding to side lengths of the squares.

Instruction:

- Ask the students to combine the straws to create triangles without right angles.
- In a method similar to Amanda's manipulative, have the students put the corresponding squares on each side of the triangle. (Appendix E)
- As an exit slip, have the students record the area of the small, medium and large squares in a chart. Ask them to comment on whether or not Pythagoras' relationship holds for all triangles or not. (Assessment)
- When going over the exit slips, confirm that only right triangles have this relationship. Explain that this is the reason why in Amanda's manipulative, some of the triangles did not add perfectly, since they were not perfect right triangles.

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4. Magic triangle

Objective:

1. This is an enrichment activity to get students working with larger numbers and discovering patterns and relationships.

Materials:

Each student should have

- A copy of the handout "The Magic Triangle" (<u>Appendix F</u>)
- A calculator

Instruction:

- Ask each student to complete the given worksheet, using a calculator if necessary.
- Optional: discuss with them why the magic triangle works: if you multiply each number in a Pythagorean triplet (i.e., 3, 4, 5) by the same number, then the Pythagorean model will still hold true.

5. Area of semi-circles and other shapes

Objectives:

1. Students should understand that the Pythagorean model does not work solely for squares. Any shape, as long as the relative proportions are the same, will yield the same relationship. This reinforcement activity is designed to have students discover that semi-circles and triangles work as well.

Materials:

Each student should have:

- A compass to draw circles
- A ruler to measure
- A handout with a right triangle and midpoints on each side marked (<u>Appendix G</u>)

Instruction:

- Ask each student to complete the given worksheet (in pairs, or groups).
- The semi-circle example is provided on the worksheet. Earlier this year, they will have determined how to take the area of a circle. If necessary, review the equation with them.
- If students are stuck, provide hints they learned to find the area of the other shape earlier this year (triangle).
- Once they realize that it is a triangle, you may want to remind them of the properties of an equilateral triangle (that all sides are equal in length, and has 60° angles).
- The "height" of the triangle can be determined solely through measuring with their ruler. This may result in minor precision errors. Encourage students to draw their triangles as accurately as possible, and even measure the base and height in millimeters.
- Teacher's note: this relationship will also hold true for more complex shapes (pentagons, hexagons, etc...), as long as each side is of the same length.

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6. Word problems (create their own as well!)

Objective:

1. To have students apply the Pythagorean model to word problems

Instruction:

- Below are some sample problems. It is important to remember when creating your own problems that students are not able to take square roots unless the numbers are common perfect squares. Also, remember the emphasis is on using the model, not on the formula.
- After students have completed word problems, have them create their own either to trade with a partner or to make a class problem set. Collect these for Assessment.

Sample Problems:

1. You are designing a new pair of 3-inch heels. In order to be able to balance and maintain comfort, you need to have a distance of 4 inches from the heel to the point where the ball of the feet meets the ground. How many inches must the arch of the shoe be between the ball of the foot and the heel?

2. I'm at Future Shop when I see a TV on sale for a blowout price. It's 100-cm wide (I remember that screen diameters are measured diagonally) and 60 cm tall. I know that my TV stand is 75 cm wide by 60 cm tall. Will this great deal fit into my current TV stand or will I have to buy a new one?

3. (a) Some ambitious students at our school have raised enough money to build a skate park. We've decided that we would like a 10 m ramp, and in order to get the angle we want, the ramp needs to extend 8 m from the pillar we are building it off of. How tall must the pillar be? We already know that the base of the pillar will be 6.5 m.



3. (b) It's your lucky day, and the school decides to let you decorate your skate park with spray paint! If one can of spray paint covers 5 m^2 , how many cans will you need to graffiti the whole ramp?

3. (c) If one can of spray paint costs \$7, how much money will you have to bring? Remember sales tax!

3. (d) There is still enough material left over to build a smaller ramp from a square board. The ramp is 2.5 m along the ground and 0.7 m high. If the spray pain cans you want cover 2 m^2 and costs \$5 each, how many cans will you need to paint the ramp's surface (only the square board) and how much will it cost?

Teacher's note: It is a good idea to give students a flat net-model of the skate park ramp so that they will understand that the top of the ramp is a square. For a cutout net-model, please see <u>Appendix H</u>.

4. We've been lucky enough to be funded to extend our skate park and part of one of our extreme ramps is to build a connecting ramp between two existing large pillars. We measure the height of the first pillar to be 1 m and the height of the second to be 4 m. We also measure the distance between the pillars to be 4 m. How much board will I need to purchase to build this ramp?

7. Pythagorean tree

Objectives:

- 1. Have some fun with a neat demo!
- 2. Conduct some basic analysis of the pattern
- 3. Understand the connection between a Pythagorean tree and fractals

Materials:

Each student should have access to:

- a computer with an internet connection
- a ruler to measure
- graph paper

Instruction:

Have the students draw a large square, ~ $\frac{1}{4}$ the width of their graph paper (with the paper in a landscape configuration). Then have them draw a 45°-45°-90° right angle triangle, using the top of the square as a hypotenuse. Draw squares from the sides of the triangles. Repeat this process again – right triangle, squares, right triangle, squares, etc. This makes an isosceles Pythagorean tree. Other Pythagorean trees can be found by drawing x°-(90 – x)°-90° right angle triangles, and then the squares on each of the sides. Other trees can be drawn as well, by using a non-right-angle triangle. Please see <u>Appendix I</u>.

Have them play with the java applet found at

http://www.ies.co.jp/math/java/geo/pytreea/pytreea.html

A connection can now be made between the Pythagorean tree and fractals. Sample Questions and Activities:

- 1) What is a unique characteristic of an isosceles Pythagorean tree, as opposed to other trees you could make with the applet? Flatness, symmetry, spiral.
- 2) Why is the Pythagorean tree a fractal? Self-similar on different length scales.
- 3) Look up some examples of fractals, and find one that is similar to the isosceles Pythagorean tree. Why do you think they are so similar? Levi C-curve (see Appendix C). It maps to the outside of the tree. Uses 45-degree angles.
- 4) Can you make another pattern using this Algorithm? (e.g.) Honeycomb using equilateral triangles.
- 5) Can you find a practical use for a Pythagorean tree? What about a non-practical use? Explain. Art. Antenna due to fractal nature.

8. Choice board final assignment

Objectives:

- 1. To have students extend their knowledge of Pythagoras and his contributions to mathematics and society.
- 2. To have students draw connections between Pythagorean mathematics and other subjects, nature, and everyday life.

Materials:

• An example of a choice board project is attached (<u>Appendix J</u>), which has been differentiated based on Gardner's multiple intelligences.

Instruction:

- This enrichment activity can be used if there is sufficient time as a nice way to end a unit and provide further assessment.
- Students can work in small groups to complete one of the projects on the choice menu, which they will present to the class in the form of their choice (e.g. a poster, oral presentation).

Appendix A: Reviewing area of shapes



Appendix B: Review of Right Angle Triangles



Activity #1 – Basics of Right Angle Triangles

1. Ask students to identify the indicated parts of a right angle triangle, (i.e.) the right angle and the hypotenuse.

Find the Hypotenuse(s)!





Activity #2 – Identifying Angles & Triangles

- 2. Ask students to locate and label the hypotenuse on any right angle triangle they can find. This includes the top-left, the top-right, and the bottom-right triangles. They should demonstrate that they understand the hypotenuse is opposite of the right angle.
- 3. Have them identify other angles, such as an obtuse angle (large angle on bottom-left triangle), an acute angle (any angle less than 90 degrees), an equilateral triangle (all sides equal: centre triangle), an isosceles triangle (two sides equal: bottom-left and maybe the top-left), and a scalene triangle (no sides equal: topright and bottom-right triangles).

Appendix C1: SMARTboard Manipulative



Appendix C2: Additional Reinforcement Manipulative

Try to move the small blue square and the medium square onto the large square.



Appendix D: NCTM Manipulative



Appendix D, continued



Appendix E: Straw activity for non-right angle triangles



Exit Slip: Did you find that the Pythagorean relationship holds for all triangles?

Appendix F: The Magic Triangle

1. Complete the three magic squares. The sum of the numbers from each column, each row, and each diagonal must be the same.



- 2. Does Pythagoras' theorem apply to...
 - the numbers written in the middle (central) boxes?
 - the numbers written in other corresponding boxes?
 - the sums of numbers written in the 4 corners of each square?
 - any other combination of numbers written within the squares?

Appendix G: Investigation of Other Pythagorean Models



Using one of the triangles below, determine if the Pythagorean model holds true for semicircles. What other shapes do you think would work? What other shapes do you know how to determine the area of?



If you think you have found another model, try using it in different situations to see if it holds true for all right-angled triangles.

Appendix H: Flat net-model of skate park ramp



Appendix H, continued



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Appendix I: Pythagorean Tree as a Fractal



Notice the self-similar nature of the Pythagorean tree, as compared to the naturally occurring fractal in Romanesco Cauliflower.

Below is the Levy C Curve compared to the isosceles Pythagorean tree. Note the similarities. Look up how the Levy C Curve is generated, and compare that to an isosceles Pythagorean tree.



<u>Appendix J</u> Final Project Choice Board

Choice #1 (spatial) Study how the golden ratio/golden rectangle has been used in art and architecture. Provide the class with examples. Create your own work of art or a model of an architectural structure that	Choice #2 (musical) Research and explain how Pythagoras used math ratios to develop the musical scale. Construct your own stringed instrument and use these ratios to create a musical	Choice #3 (logical/mathematical) Research a current technology that can track its history back to Pythagoras's theory. Construct a visual timeline highlighting the changes it has undergone to get to where it is
follows this ratio and share it with the class.	scale.	today.
Choice #4 (interpersonal) Interview a carpenter about using the 3-4-5 rule and why it is an important tool. Video record or transcribe the interview and summarize the important findings to share with the class.	Choice #5 Your choice! (Check your idea with the teacher first)	Choice #6 (verbal/linguistic) Research the life of Pythagoras. Draw connections between the social influences of his time (religion, politics, art, etc.) and his theories. Share his biography with the class.
Choice #7 (naturalist) Study where and how the golden ratio/spiral is found in nature. Collect some examples to share this principle with the class.	Choice #8 (intrapersonal) Create an "All About Me" scrapbook for Pythagoras. Include photographs and captions of contributions you think he would have been proud of in his life.	Choice #9 (bodily/kinesthetic) Present a role play as Pythagoras presenting one of his contributions to the class and put together an Artifact Box of several objects illustrating this concept to use in the presentation.

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Pythagorean Theorem

Grade: 8 Unit: Patterns and Relations SCO: Students should be able to demonstrate an understanding of the Pythagorean relationship, using models.

Materials (for each pair of students):

- One square piece of grid paper in each of the following sizes (appendix A):
 - o
 1 cm x 1 cm
 o
 6 cm x 6 cm
 o
 11 cm x 11 cm

 o
 2 cm x 2 cm
 o
 7 cm x 7 cm
 o
 12 cm x 12 cm

 o
 3 cm x 3 cm
 o
 8 cm x 8 cm
 o
 13 cm x 13 cm

 o
 4 cm x 4 cm
 o
 9 cm x 9 cm
 o
 14 cm x 14 cm

 o
 5 cm x 5 cm
 o
 10 cm x 10 cm
 o
 15 cm x 15 cm
- Handout of right angle (appendix B)
- Scotch tape
- Handout of spreadsheet (appendix C)
- Handout of puzzle (appendix D)
- Scissors

Advance preparation:

- Cut out one set of paper grids for each pair (appendix A), marking the area of each on the back
- Place each set of grids in a plastic zip-lock bag.
- Print one copy of the right angle (appendix B), one copy of the spreadsheet (appendix C), and one copy of the puzzle (appendix D) for each pair of students.
- Have a quick look at appendix E to see the solution to the puzzle you will be handing out.

Instructions:

- Try to introduce the lesson in a way that provides context for the students. You may use any suitable real-life example, but I will use that of the 3-4-5 rule in carpentry. Explain to the students that carpenters need to make sure that some things that they are building have right angles. They have a precise way of checking angles called the 3-4-5 rule. They measure 3 units from the corner in one direction, and 4 units in the other direction from the corner. They then draw a line connecting the points to form a triangle. If the line measures 5 units then they know that they have made a right angle. Today we will be discovering why this rule works.
- Start the lesson by reminding students about right angles and right angle triangles (Draw a picture of a right angle triangle on the board). Explain that the two shorter sides of the right triangle are denoted as "a" and "b", and that the longest side (the hypotenuse) is denoted as "c"

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- Explain that today we will be looking for a relationship that describes the area of squares which have the sides of the right triangle as their bases. Draw a square on each side of the triangle that you drew previously on the board.
- Review with the students how to find the area of a rectangle (base x height) and that the area of a square would be base². This is a good chance for them to demonstrate previous knowledge so rather than giving them the formula, ask if anyone recalls it.
- Distribute the materials (except for the puzzle) to the class and ask them to start working in pairs by taping the handout with the right angle to their table. Ask them how do they know that this is a right angle? Could this easily be turned into a right triangle? How?
- Explain that they will be using the square grids to construct right triangles of different sizes and that when they think they have found 3 squares that match up to form a right triangle, they can record the areas of the squares in the appropriate place on the handout (appendix c). To make sure that their triangles are right triangles, they should construct them by lining the first 2 squares up on the right angle handout, and finding a third square to form the hypotenuse. You may show them an example using the 3-4-5 rule from the intro, explaining that when squared, this makes 9-16-25 (tie back in to carpentry example).
- Once each pair of students has found several combinations or Pythagorean triplets, have them read them aloud while you write them on the board or else have them come up and fill in a table on the board.
- Once this is up where everyone can clearly see the results, ask if anyone sees a pattern? If there is no response, ask if they see a pattern specifically between a,b and c. Someone should notice that when we add the values from column a and b, we get the values from column c. This is the Pythagorean model: the area of the square on side a plus the area of the square on side b equals the area of the square on side c.
- Did this pattern hold true for each triplet found by the students? If not, then explain that those cases were anomalies (the squares may have shifted so that the triangle was no longer a right triangle). To prove to them that this pattern is true, have each pair of students cut out the pattern on appendix d, cut out the square marked 5 (square a), cut out the square with the dotted lines, and cut on the dotted lines to get the shapes 1 through 4 (square b). Using these shapes, they can assemble them to fill in the area of square c (the largest).
- If they are struggling to complete the puzzle, help them by suggesting that they place piece # 5 in the middle, and fill the others around it. After they are done, discuss the pattern with them again. Ask them what they have done with squares a and b. They should agree that squares a and b have both fit into square c, or in other words, that square a plus square b is the same size as square c.
- Ask the students "Do you think that this relationship would hold true if we used half-circles instead of squares?" "What about any other geometrical shape such as a triangle?" Either do a quick example with them of these cases on the board or else give them as problems to solve with their partner.

Enrichment

- The actual Pythagorean theorem $(a^2 + b^2 = c^2)$ is not introduced until grade 10, but if you have students who seem to have a very good grasp on the concept of the Pythagorean relationship, you may offer this website as enrichment: <u>http://arcytech.org/java/pythagoras/index.html</u>
- If you do not wish to send the students to the website on their own, you can print out templates from the website and use them as an in-class activity.

Appendix C

The Pythagorean Relationship

Names:

Area of square on side "a"	Area of square on side "b"	Area of square on side "c"

Based on the data collected above, can you put into words the Pythagorean relationship?



